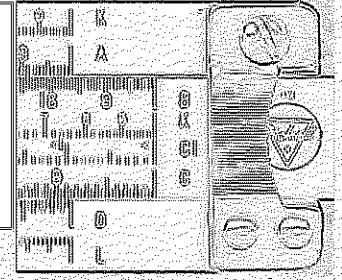
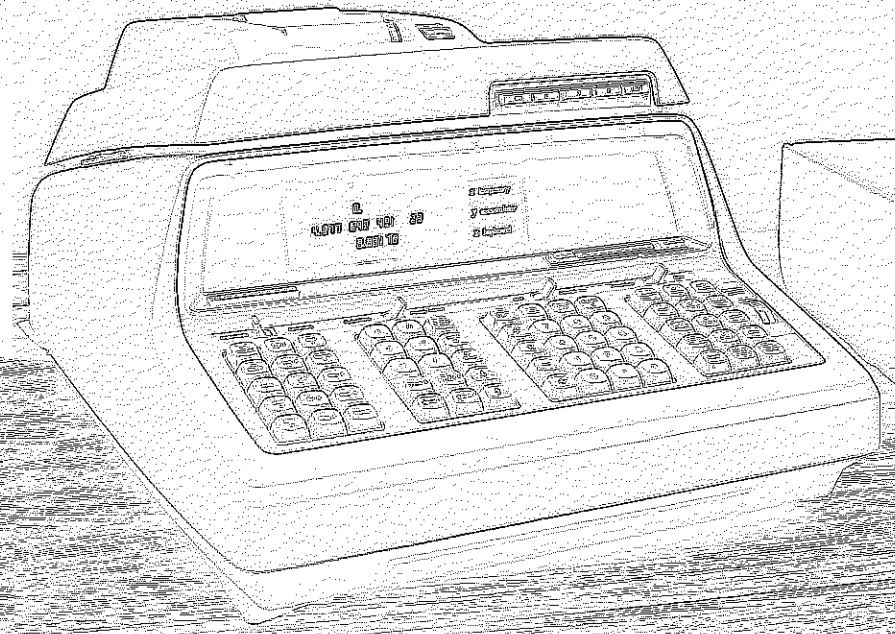
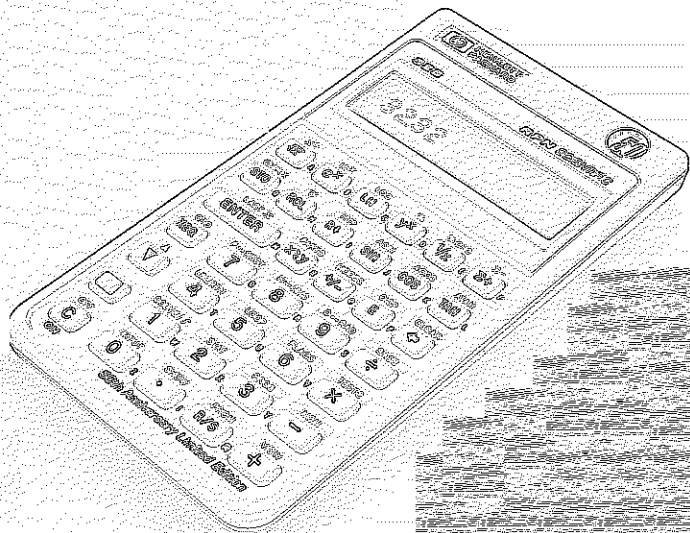
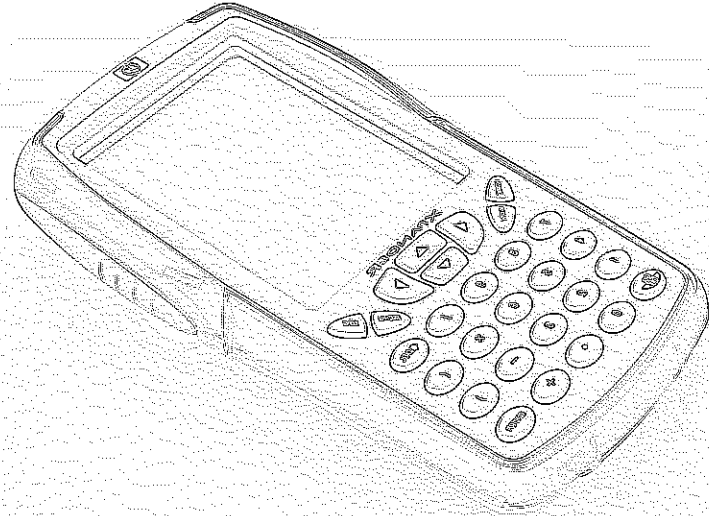
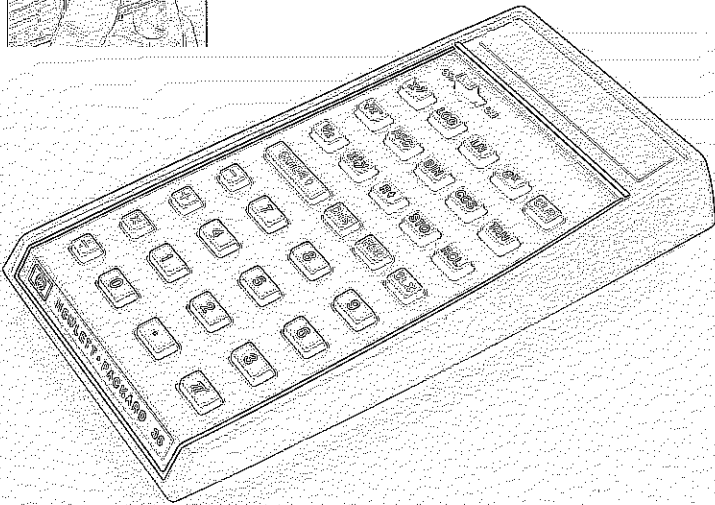
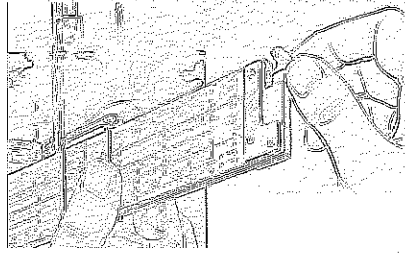
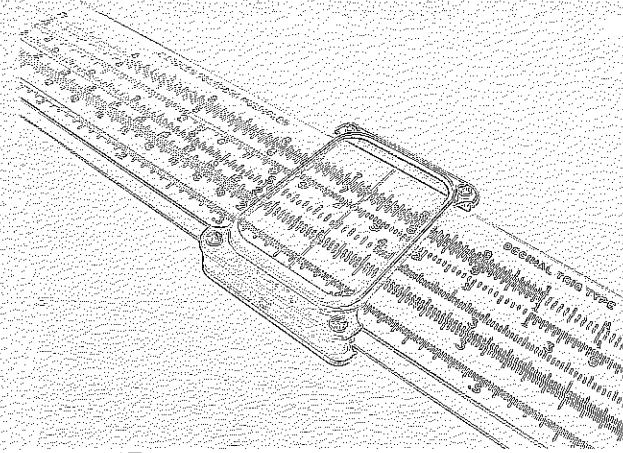
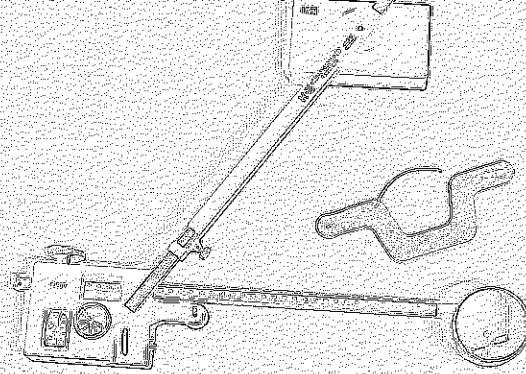
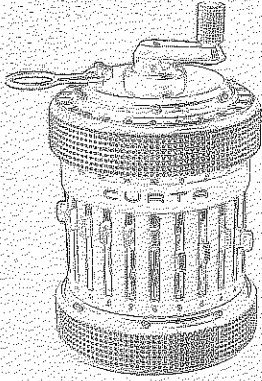


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**2004 UIL "A" Release Tests**  
**Mathematics - 9 pages**





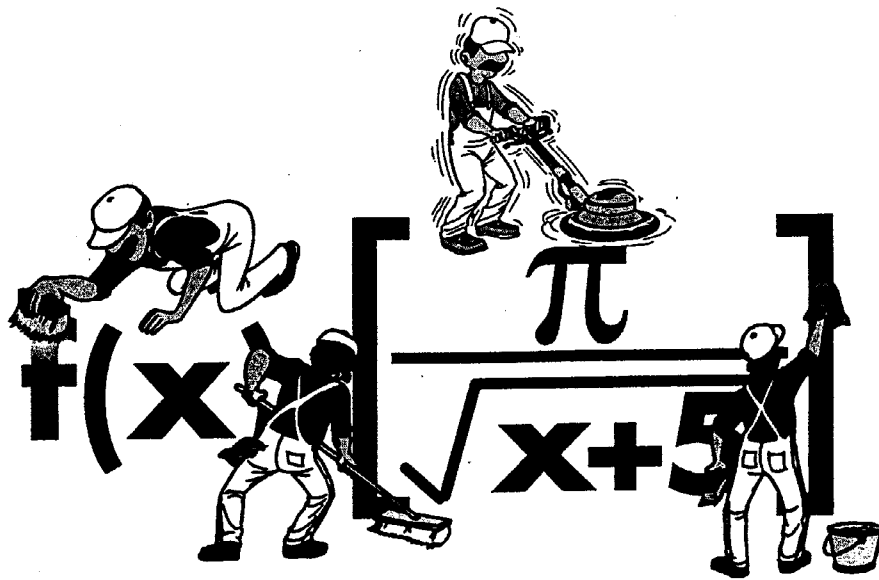
**UNIVERSITY INTERSCHOLASTIC LEAGUE**

*Making a World of Difference*

# Mathematics

## INVITATIONAL A

### Spring 2004



**WRITE ALL ANSWERS WITH  
CAPITAL LETTERS**

**DO NOT TURN THIS PAGE UNTIL  
YOU ARE INSTRUCTED TO DO SO!**

1. Evaluate:  $20 \div 15 - (3 + 6) \times 8^2$
- (A)  $-490\frac{2}{3}$  (B)  $-574\frac{2}{3}$  (C)  $1201\frac{7}{9}$  (D)  $277\frac{1}{3}$  (E)  $76\frac{1}{3}$
2. Music City is having a special sale if you buy 3 CD's. The first one is 25% off. The second one is  $33\frac{1}{3}\%$  off. And the third one is 50% off. What would the total cost of the 3 CD's be before taxes if the regular price of a CD is \$18.48? (to the nearest cent)
- (A) \$ 20.02 (B) \$ 27.18 (C) \$ 32.32 (D) \$ 35.42 (E) \$ 45.43
3. If  $5 - (4 - x) = 3(4 + x)$  then  $4x$  equals:
- (A)  $-26$  (B)  $-22$  (C)  $-5.5$  (D) 11 (E) 13
4. If  $\frac{1}{2y} - \frac{3}{4x} = \frac{5}{6}$ , then  $y$  equals \_\_\_\_\_.
- (A)  $\frac{6x}{10x+9}$  (B)  $\frac{6x}{10+9x}$  (C)  $\frac{6x}{10x-9}$  (D)  $\frac{7x-9}{8}$  (E)  $\frac{7x}{8}$
5. A right isosceles triangle has an area of  $6.125 \text{ in}^2$ . The length of hypotenuse is: (nearest tenth).
- (A) 2.5 in (B) 3.5 in (C) 4.9 in (D) 7 in (E) 12.3 in
6. The sum of the interior angles of a regular convex polygon is 900 degrees. The polygon is a \_\_\_\_.
- (A) Heptagon (B) Dodecagon (C) Hexagon (D) Octagon (E) Nonagon
7. Evaluate:  $\sum_{x=1}^4 (-1)^x (1-x)$
- (A)  $-2$  (B)  $-1$  (C) 0 (D) 4 (E) 6
8. Let  $f(x) = 4 - 3x$  and  $g(x) = 3 + 5x$ . If  $h(x)$  is the inverse function of  $\frac{f(x)}{g(x)}$ , find the value of  $h(-1)$ .
- (A)  $-\frac{1}{8}$  (B)  $-3\frac{1}{2}$  (C)  $-\frac{1}{2}$  (D)  $\frac{7}{8}$  (E)  $\frac{2}{7}$
9. Simplify:  $\sin\theta + \cos\theta \cot\theta$
- (A)  $2\sin\theta$  (B)  $\cos^2\theta$  (C)  $\sec\theta - \sin\theta$  (D)  $\tan\theta$  (E)  $\csc\theta$
10. A ramp rises 1" for every 12" of run. Find the angle the end of the ramp makes with the ground when the ramp is 8' long. (nearest tenth of a degree)
- (A)  $38.4^\circ$  (B)  $7.1^\circ$  (C)  $4.8^\circ$  (D)  $0.6^\circ$  (E)  $8^\circ$

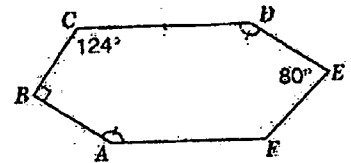
11. Find the remainder when  $f(x) = 2x^3 - 3x^2 + 4x - 7$  is divided by  $x - 2$ .
- (A) 40      (B) 16      (C) 5      (D)  $-2$       (E)  $-43$
12. Find the determinant of the  $2 \times 2$  matrix  $\begin{bmatrix} -3 & 1 \\ -1 & 3 \end{bmatrix}$
- (A)  $-8$       (B)  $-10$       (C) 0      (D) 8      (E) 10
13. Determine the limit:  $\lim_{x \rightarrow 1} \frac{x-1}{x^2-1}$
- (A) 0      (B) .5      (C) 1      (D) 2      (E) does not exist
14. How many horizontal tangents does the curve  $y = x^4 - 9x^2 + 3$  have?
- (A) 0      (B) 1      (C) 2      (D) 3      (E) 4
15. The probability of scoring above 200 on this test is  $\frac{3}{8}$ . If 24 students scored above 200 then how many students took this test?
- (A) 9      (B) 25      (C) 64      (D) 72      (E) 99
16. Coach Bratton has 3 seniors, 4 juniors, 2 sophomores, and 5 freshmen on her team. She wants to put them in teams where each team consists of one member from each of the grade levels. How many different 4 member teams can she make?
- (A)  $2!$       (B)  $3!$       (C)  $4!$       (D)  $5!$       (E)  $6!$
17. An operation " $\triangle$ " is defined by:  $a \triangle b = a^b - b^a$ . What is the value of  $(2 \triangle 3) \triangle (3 \triangle 2)$ ?
- (A)  $-2$       (B) 0      (C) 1      (D) 3      (E) undefined
18. The terms of the sequence 3,5,7,11,13,17,... represent the length of the shortest leg of a primitive Pythagorean triple. Find the length of the hypotenuse of the right triangle for the next term.
- (A) 177      (B) 179      (C) 181      (D) 183      (E) 185
19. If  $3x^2 = 4x - 5$  and  $r_1$  and  $r_2$  are the roots, then  $r_1 \times r_2$  equals \_\_\_\_\_.
- (A) 0.6      (B)  $1\frac{2}{3}$       (C) 0.8      (D)  $-1\frac{1}{4}$       (E)  $1\frac{1}{3}$
20. If  $\frac{1}{25}$  of  $5^{20}$  is  $125^x$ , then the value of  $x$  is:
- (A)  $-3$       (B)  $5^3$       (C)  $-5$       (D) 6      (E)  $2^3$

21. If the exterior sides of two adjacent acute angles are perpendicular, then the angles are always \_\_\_\_\_.

- (A) vertical (B) congruent (C) complementary (D) transversal (E) supplementary

22. Determine the measure of  $\angle AFE$  if  $\overline{CD} \parallel \overline{AF}$ .

- (A)  $124^\circ$  (B)  $130^\circ$  (C)  $134^\circ$  (D)  $146^\circ$  (E)  $150^\circ$



23. A three ring dart board has a center ring worth 5 points, an outer ring worth 1 point and the middle ring worth 3 points. The probability of hitting the center ring is 23%, the outer ring is 48% and the middle ring is 29%. What is the mathematical expectation on any one dart?

- (A) 3 (B) 1.75 (C) 4.25 (D) 1 (E) 2.5

24. Paul Perfect has taken 6 tests and has a test average of 86. What will his average be if he scores 100 on his next test?

- (A) 87 (B) 88 (C) 89 (D) 90 (E) 91

25. The polar coordinate  $(-3, 135^\circ)$  is converted to the rectangular coordinate  $(x, y)$ . Find  $x$ . (to the nearest hundredth)

- (A)  $-.71$  (B) 2.12 (C) 2.29 (D)  $-3.71$  (E) .33

26. Three and one fifth billion times two and one fourth million plus five and one eighth trillion has how many consecutive zeros when written out using digits instead of words?

- (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

27. If 110% of A is  $\frac{5}{8}$  of B, then A is what per cent of B?

- (A) 176 (B)  $43\frac{2}{9}$  (C)  $47\frac{1}{2}$  (D) 76 (E)  $56\frac{9}{11}$

28. If  $n \geq 1$ , then  $a_{n+1} = a_n + f(n)$  defines the sequence 7,10,17,28,43, ... . Find an expression for  $f(n)$ .

- (A)  $2n + 1$  (B)  $3n + 2$  (C)  $4n - 1$  (D)  $5n - 2$  (E)  $6n - 3$

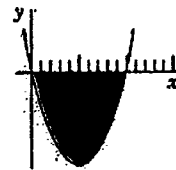
29. Find the solution set:  $2 + 5x = |14 + 3x|$

- (A)  $\{-2, 6\}$  (B)  $\{-2\}$  (C)  $\{6\}$  (D)  $\{-6, 2\}$  (E)  $\{2, 6\}$

30. When  $k$  is divided by 5, the remainder is 3. What is the remainder when  $2k$  is divided by 5?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

31. The graph of  $-y^2 = 6 - x^2$  is a(n):
- (A) parabola      (B) circle      (C) ellipse      (D) hyperbola      (E) line
32. If  $a_1 = 1$ ,  $a_2 = 1$ , and  $a_n = a_{n-2} + a_{n-1}$  for  $n \geq 3$ , then  $a_7$  equals:
- (A) 3      (B) 8      (C) 21      (D) 34      (E) 13
33. Farmer Frank has a sheet of metal that is 16 feet by 10 feet. He is going to cut out the same size square from each of the four corners, then fold up the sides, solder them, and make a rectangular cistern. How long must the side of each of the squares be if the cistern is to hold the greatest possible amount of water?
- (A) 2 feet      (B) 18 inches      (C) 1 foot      (D) 6 inches      (E) 2.5 feet
34.  $\angle ABC$  is inscribed in a circle. If the measure of the intercepted arc is  $135^\circ$  then  $m\angle ABC$  is:
- (A)  $45^\circ$       (B)  $67.5^\circ$       (C)  $90^\circ$       (D)  $225^\circ$       (E)  $270^\circ$
35. Hickory, Dickory, and Doc looked up at the circular clock. The time shown was 9:04 PM. Find the measure of the smaller angle in degrees between the big hand and the little hand.
- (A)  $22^\circ$       (B)  $48^\circ$       (C)  $68^\circ$       (D)  $112^\circ$       (E)  $158^\circ$
36. Farmer John's right cylindrical tank is 20% full. The tank is 8 feet high and has a diameter of 12 feet. How many gallons will he have to add to the tank to fill it to the top? (nearest gallon)
- (A) 21,658      (B) 1,354      (C) 6,768      (D) 17,326      (E) 5,415
37. Let  $y = \arcsin \theta$ . Which of the following ranges will produce a function graph?
- (A)  $[0, \pi]$       (B)  $[-\infty, \infty]$       (C)  $[-\pi, \frac{\pi}{2}]$       (D)  $[-\frac{\pi}{2}, \frac{\pi}{2}]$       (E)  $[-\pi, \pi]$
38. A circle with a center at the origin has a radius of 3 units. If you start at  $(0,3)$  and travel  $\frac{5\pi}{2}$  radians, where will you come to a stop?
- (A) x-axis      (B) QI      (C) QIII      (D) QIV      (E) y-axis
39. Simplify:  $\frac{(n-1)! - (n-2)!}{n!}$
- (A)  $\frac{n+1}{n}$       (B)  $\frac{n-1}{n(n-2)}$       (C)  $\frac{n-1}{n-2}$       (D)  $\frac{n-2}{n(n-1)}$       (E)  $\frac{n^2 - n + 1}{n-1}$
40. The arithmetic mean of 87.2 and 27.8 is \_\_\_\_\_ % of the geometric mean of 87.2 and 27.8.
- (A) 116.8      (B) 115.0      (C) 85.6      (D) 57.5      (E) 49.2



41. The equation of the parabola shown is  $y = x^2 - 10x$ . Its x-intercepts are (0,0) and (10,0) and its vertex is (5, -25). Find the shaded area in square units.
- (A)  $55\frac{1}{2}$       (B)  $93\frac{3}{4}$       (C) 125      (D)  $166\frac{2}{3}$       (E) 200
42. If 3 geese cost as much as 4 ducks and 5 ducks cost as much as 6 storks, what is the cost of a stork if a goose costs \$2.00?
- (A) \$1.00      (B) \$6.00      (C) \$1.50      (D) \$7.50      (E) \$1.25
43. 70 miles per hour equals \_\_\_\_\_ feet per second.
- (A)  $34\frac{2}{9}$       (B)  $56\frac{1}{3}$       (C)  $102\frac{2}{3}$       (D) 308      (E)  $513\frac{1}{3}$
44. The set of odd numbers is closed under \_\_\_\_\_.
- (A) addition      (B) subtraction      (C) multiplication      (D) division      (E) square root
45. Herr Euler's polyhedron has 5 faces and 9 edges. How many vertices does it have?
- (A) 4      (B) 6      (C) 7      (D) 8      (E) 14
46. If  $f(x) = 4x - 1$  then  $f^{-1}[f(f(3))]$  equals:
- (A) 11      (B) -13      (C) 10.5      (D) -3.25      (E) 0.5
47. If the  $\sin x^\circ = .6820$  and the  $\sin(x + 1)^\circ = .6947$ , then  $\sin(x^\circ + 15')$  is:
- (A) .6835      (B) .6915      (C) .6902      (D) .6865      (E) .6852
48. Find the digit in the thousandth place of the series  $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$ , when  $x = 3$ .
- (A) 3      (B) 5      (C) 8      (D) 2      (E) 0
49. Evaluate:  $\prod_{n=2}^5 (-1)^{n-2} + n$
- (A) 10      (B) 14      (C) 24      (D) 80      (E) 120
50. Simplify:  $\left[\frac{a}{b} + \frac{1}{a}\right] \times \frac{b}{a} \div \frac{1}{a}$
- (A)  $a + \frac{b}{a}$       (B)  $\frac{a^2 + b}{a^3}$       (C)  $\frac{a^3 + b}{b^2}$       (D)  $\frac{a}{b} + b$       (E) 1

51. Line  $m$  is perpendicular to line  $n$  at point  $(1, 7)$ . Point  $(3, -3)$  lies on Line  $n$ . Which of the following points lies on line  $m$ ?
- (A)  $(6, -8)$  (B)  $(-1, 7)$  (C)  $(-4, 6)$  (D)  $(4, -4)$  (E)  $(2, -10)$
52. Kandy Barr wants to mix some sugar candy worth \$1.90 a pound with sour candy worth \$1.65 a pound. How many pounds of sour candy will she need to make a mixture of sweet and sour candy that weighs 100 pounds and sells for \$1.75 a pound?
- (A) 25 (B) 40 (C) 50 (D) 60 (E) 75
53. What value of  $c$  will make  $z^2 + 1.2z + c$  a trinomial square?
- (A) 3.6 (B) 1.44 (C) .06 (D) .09 (E) .36
54. A pyramid with an equilateral triangle as its base has volume  $V$ . The side of the triangle is  $2S$ . Find the height of the pyramid.
- (A)  $\frac{\sqrt{3}V}{4S^2}$  (B)  $\frac{2\sqrt{3}V}{3S^2}$  (C)  $\frac{\sqrt{3}V}{2S^2}$  (D)  $\frac{\sqrt{3}V}{3S^2}$  (E)  $\frac{\sqrt{3}V}{S^2}$
55. Which of the following functions has an inverse function in the  $xy$  plane?
- (A)  $y = -4$  (B)  $y = |3x|$  (C)  $y = x^2$  (D)  $y = x^4$  (E)  $y = \sqrt{x-4}$
56. Which of the following equations can be obtained from the graph of the parent function  $y = \cos x$  by applying a vertical stretch of 2 units and a vertical shift of 3 units?  $y =$
- (A)  $3 + 2\cos x$  (B)  $-3 + 2\cos x$  (C)  $2 - 3\cos x$  (D)  $2 + 3\cos x$  (E)  $-2 - 3\cos x$
57. Which of following is NOT a solution for  $\cos \theta + 2 = 3 \cos \theta$ ?
- (A) 0 (B)  $\pi$  (C)  $2\pi$  (D)  $-2\pi$  (E)  $4\pi$
58. If  $5^{x+y} = 6$  and  $5^{x-y} = 4$ , then  $25^x =$ ?
- (A)  $2\sqrt{6}$  (B) 10 (C) 20 (D) 24 (E) 125
59. Find the first term of the arithmetic sequence:  $a, 40, b, c, 19, \dots$
- (A) 49 (B) 47 (C) 45 (D) 43 (E) 41
60.  $\int (\sec^2 x) dx = \underline{\hspace{2cm}} + C$ , where  $C$  is some arbitrary constant.
- (A)  $\frac{\sin x}{\cos x}$  (B)  $-\frac{\cos x}{\sin x}$  (C)  $1 - \tan^2 x$  (D)  $\csc^2 x$  (E)  $-\sec^2 x$

**University Interscholastic League  
MATHEMATICS CONTEST  
HS • Invitational A • 2004**

**ANSWER KEY**

- |     |   |     |   |     |   |
|-----|---|-----|---|-----|---|
| 1.  | B | 21. | C | 41. | D |
| 2.  | D | 22. | C | 42. | E |
| 3.  | B | 23. | E | 43. | C |
| 4.  | A | 24. | B | 44. | C |
| 5.  | C | 25. | B | 45. | B |
| 6.  | A | 26. | E | 46. | A |
| 7.  | A | 27. | E | 47. | E |
| 8.  | B | 28. | C | 48. | B |
| 9.  | E | 29. | C | 49. | E |
| 10. | C | 30. | B | 50. | A |
| 11. | C | 31. | D | 51. | C |
| 12. | A | 32. | E | 52. | D |
| 13. | B | 33. | A | 53. | E |
| 14. | D | 34. | B | 54. | E |
| 15. | C | 35. | D | 55. | E |
| 16. | D | 36. | E | 56. | A |
| 17. | A | 37. | D | 57. | B |
| 18. | C | 38. | A | 58. | D |
| 19. | B | 39. | D | 59. | B |
| 20. | D | 40. | A | 60. | A |